**Composite Function**

Performing algebraic operations on functions combines them into a new function, but we can also create functions by composing them. When the output of one function is used as the input of another, the resulting function is known as a composite function. We represent this combination by the following notation:

The left-hand side is read “ “ and the right-hand side is read “”

Another way to think about composite functions is as follows:

Here we see that we start with input give it to the function , the function gives us the output which we then give to the function as input, and finally gives us the output .

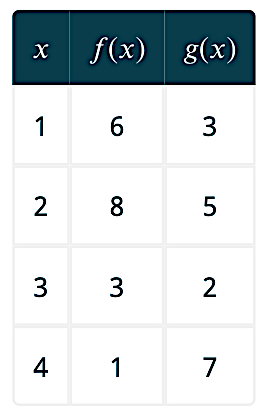
Example 1: Consider the functions:

1. Determine
2. Determine
3. Determine

**Evaluating Composite Functions with Tables and Graphs**

When working with functions given as tables, we read input and output values from the table entries and always work from the inside to the outside. We evaluate the inside function first and then use the output of the inside function as the input to the outside function.

Example 2: Use the table to determine the following.



Example 3: Use the graphs to determine the following.

A graph with a line drawn on it

Description automatically generated

1. c.
2. d.

**Domain of a Composite Function**

The domain of a composite function is the set of input values such that

* is in the domain of
* is in the domain of

Example 4: Consider the functions:

1. Determine the domain of
2. Determine the domain of

**Inverse Functions**

For any one-to-one function , we represent its inverse as , read as “inverse of ”. The raised is part of the notation. It is not an exponent; it does not imply a power of . In other words, does NOT mean because is the reciprocal of and not the inverse.

Using composite notation, inverse functions have the following property:

The outputs of the function are the inputs to , so the range of is also the domain of Likewise, because the inputs to are the outputs of the domain of is the range of

A diagram of a complex equation

Description automatically generated

Example 5: Given and , determine if

Example 6: Find the inverse of the one-to-one function,

Now that we can find the inverse of a function, we will explore the graphs of functions and their inverses. The graph of is the graph of reflected about the diagonal line which we will call the identity line. The visual below demonstrates this uses the quadratic function on the restricted domain of Note:

A graph of a function

Description automatically generated

Example 7: Sketch the graph of given the graph of below.

A graph of a function

Description automatically generated

**Transformations**

Often when given a problem, we try to model the scenario using mathematics in the form of words, tables, graphs, and equations. One method we can employ is to adapt the basic graphs of the parent functions to build new models for a given scenario. There are systematic ways to alter functions to construct appropriate models for the problems we are trying to solve.

One simple kind of transformation involves shifting the entire graph of a function up, down, right, or left.

The simplest shift is a **vertical shift**, moving the graph up or down, because this transformation involves adding a positive or negative constant to the function. In other words, we add the same constant to the output value of the function regardless of the input.

For a function the function is vertically shifted units. If is positive, the graph will shift up. If is negative, the graph will shift down.

**A graph of function on a grid

Description automatically generated**

We just saw that the vertical shift is a change to the output, or outside, of the function. We will now look at how changes to input, on the inside of the function, change its graph and meaning. A shift to the input results in a movement of the graph of the function left or right in what is known as a **horizontal shift.**

For a function the function is horizontally shifted units. If is positive, the graph will shift right. If is negative, the graph will shift left.

**A graph of function on a grid

Description automatically generated**

Example 8: Given , sketch the graph of

Another transformation that can be applied to a function is a reflection over the x- or y-axis. A **vertical reflection** reflects a graph vertically across the–axis, while a **horizontal reflection** reflects a graph horizontally across the –axis.

A diagram of a function

Description automatically generated

Given a function a new function is a vertical reflection of the function , sometimes called a reflection about (or over, or through) the–axis.

Given a function a new function is a horizontal reflection of the function , sometimes called a reflection about the –axis.

A function is called an **even function** if for every input

The graph of an even function is symmetric about the –axis.

A function is called an **odd function** if for every input

The graph of an odd function is symmetric about the origin.

Given a function , a new function where is a constant, is a vertical stretch or vertical compression of the function

* If , then the graph will be stretched by a factor of .
* If , then the graph will be compressed by a factor of .
* If , then the graph will have a vertical reflection and a vertical stretch or compression.

A diagram of a normal distribution

Description automatically generated

Given a function , a new function where is a constant, is a horizontal stretch or horizontal compression of the function

* If , then the graph will be compressed by a factor of .
* If , then the graph will be stretched by a factor of .
* If , then the graph will have a horizontal reflection and a horizontal stretch or compression.

A graph of a function

Description automatically generated

Example 9: Given the graph of below, sketch the graph of

A graphing of a function

Description automatically generated

Example 10: Given , sketch the graph of .